Callable or Convertible Debt? A Debt Overhang Matter

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This version: December 4, 2019

Abstract In this paper, we study how a firm chooses between issuing callable or convertible debt. To this end, we set up a dynamic model in which the firm can decide to finance itself with either callable or convertible bonds. We link the financing choice to the debt overhang problem and study the timing of calling or converting the respective bonds. Furthermore, we test our model predictions on data on corporate bonds. We find that firms that are more exposed to debt overhang issue callable rather than convertible debt. Furthermore, compared to convertible bonds, firms issue callable bonds with a higher coupon, and hold the bond for a longer period before conversion.

Keywords: Capital structure, Debt choice, Investment timing, Growth options, Effective maturity

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1 Introduction

The U.S. corporate bond market is massive, with a principal amount of more than \$9.3 trillion by 2018.⁴ While corporations issue bonds as a source of external financing, they also frequently repurchase their debt. In 2010, total cash repurchases of publicly traded debt by U.S. firms reached \$88 billion (Julio, Julio). The repurchase of corporate bonds can occur via different methods. While open market repurchases enable the issuing firm directly to buy back bonds from the secondary market, bond contracts can also have special features for repurchase. With *callable debt*, the firm offers a premium to bondholders in exchange for the repurchase option of their claim. With *convertible debt*, investors have the option to exchange firms' debt upon which firms provides a debt-for-equity exchange.

In this paper, we examine factors that influence a firms' issuance choice between callable or convertible debt. To this end, we model the optimal design of debt contracts. We use this model to examine how the choice of debt contract relates to bond and firm characteristics. We find that callable debt is more valuable for firms that are more exposed to debt overhang. Furthermore, compared to debt with convertible features, callable debt is issued with a higher coupon and held for a longer period. Using a sample of U.S. corporate bonds, we subsequently confirm our model's implications.

To study the choice of debt contract, we set up a dynamic model. A firm chooses between issuing debt with either callable or convertible features. It may invest in a second project which, if undertaken, will increase its payoff. To study the effect of debt overhang on the initial financing choice, we assume that the firm financed this second investment with equity only. Thus, when the debt overhang is more pronounced, debt holders will be able to extract a large amount of wealth from this investment.

⁴Based on data from the Securities Industry and Financial Markets As-sociation (SIFMA).See http: //sifma.org/research/statistics.aspx.

Our model offers several testable implications. First, debt with callable features is more valuable for firms that are more exposed to debt overhang. To mitigate the effect of debt overhang, the firm tends to issue a debt contract which the firm can control ex-post. Second, firms issue callable debt with a higher coupon than convertible debt. Since holders of convertible debt have the option to convert their claim into equity, we expect to compensate holders of callable debt by a higher coupon. Third, the holding time of callable bonds is longer than that of convertible bonds. Thus, the time between issuance and conversion for convertible bonds are shorter compared to the time between issuance and repurchase of callable bonds.

We test our model's predictions using a detailed sample of U.S. corporate bonds between 1990 and 2018. Using a measure of debt overhang that was recently developed by Alanis et al. (2018) we link this to the probability of issuing callable rather than convertible debt. In line with our model implications, we find that firms which are more exposed to debt overhang issues callable debt. Also, callable debt is issued with a larger coupon and held for a longer period compared to convertible bonds. We further document that firms issue callable debt with longer maturity and a higher offering amount compared to convertible bonds. Related to firm characteristics, we find that, large firms with a higher return on assets and a large Tobin's Q are more likely to issue callable bonds. In terms of coupon payments callable debt is more expensive than convertible debt. Thus larger more profitable firms are more likely to be able to pay for issuing a bond which can be expost controlled by the firm. On the other hand, firms with more cash are more likely to issue convertible bonds. This correspond to Jensen (1986) who argues that firm use convertible bonds to avoid free cash flow problems.

Debt repurchases have been studied extensively in the previous literature. Mao and Tserlukevich (2015) build a model of a firm repurchasing its corporate debt and find that costly bankruptcy encourages repurchase while taxation and transaction costs discourage repurchase. The expected gain from repurchase increases with the risk of default, and thus, with a high risk of default, there is a more significant probability that the debt holders will give concessions. Our model framework incorporates similar features but considers both the issuance and repurchase of callable or convertible corporate debt.

In our model framework, the debt overhang problem implies that equity holders call outstanding debt upon investment. Previous literature considers a similar case. Julio (Julio) provides an investment based explanation as a motive for why firms choose to repurchase debt. The market for debt repurchases serves as a substitute for renegotiation. The author finds that firms are more likely to repurchase outstanding debt when investment frictions are relatively high. This improvement is more pronounced for firms with higher expected transfers to bondholders. Frantz and Instefjord (2019) also analyze the restructuring of debt in the presence of debt overhang. In their model, a debt-for-equity exchange for removing all existing debt takes place just before investment. They find that carrying old debt over the investment threshold causes distortions to the timing of the investment decision. Finally, Kruse et al. (2014) document an investment based explanation for debt repurchases. The authors study empirically the decision to repurchase debt and examine the market reaction to announcements of offers to repurchase outstanding debt. Companies repurchase debt to circumvent restrictive covenants, which allows them to pursue promising investment opportunities. Our paper adds to the discussion by examining if the debt overhang problem influences the ex-ante optimal debt contract.

Hennessy and Tserlukevich (2008) analyze the choice between callable and convertible debt. The authors do so in light of taxes and moral hazard. They find that if managers can increase volatility without reducing the asset drift, callable bonds are optimal. Convertible bonds mitigate risk shifting and are thus optimal if risk-shifting reduces asset drift sufficiently. Contrary to these findings, our model predicts that callable bonds have a higher coupon compared to convertibles, and our empirical analysis confirms this finding. Moreover, we find that the holding period of callable debt is longer compared to that of convertible claims, which is again contrary to the previous findings which has found the conversion boundary for pure convertibles to be higher than the call boundary for callable bonds.

In a contemporaneous paper, Becker, Campello, Thell, and Yan (Becker et al.) shows that call features limit debt overhang by restricting value gains to corporate creditors. Further, longer maturity bonds are frequently issued with callable features. We add to these findings a detailed analysis of convertible debt and find that firms issue callable debt with a longer maturity compared to convertible debt.

The remainder of the paper is organized as follows. Section 2 presents our model framework in which we define the debt contract for callable and convertible debt. Section 3 considers how our model relates to the debt overhang problem and the timing for debt repurchase. Section 4 presents our numerical implementation of the model and present our testable hypotheses. Section 5 presents our data and empirical results. Section 6 discusses our model assumptions and possible extensions and Section 7 concludes. Proofs are provided in the Appendix, Section 8.

2 Model

We consider a firm initially set up with debt and equity. For simplicity, we assume that debt must fund an investment equal to I_0 (see, e.g. Shibata and Nishihara, 2012; Flor and Hirth, 2013). After initializing the firm, it earns a payoff of

$$dx_t = \mu x_t dt + \sigma x_t dz_t,\tag{1}$$

with initial value x_0 and where $z = (z_t)$ is a standard Brownian motion. Debt is issued as a perpetual contract with a constant coupon rate C > 0 and the firm faces a tax rate τ . We allow the firm to choose between two types of debt contracts: callable debt or convertible debt.⁵ Bankruptcy costs are denoted by α . Upon default, equity holders receive zero and debt holders take over the firm. We assume that all agents are risk neutral and evaluate cash flows with the constant risk-free interest rate r where $\mu < r$.

Once the initial capital structure is in place, the equity holders have a perpetual option to exploit a growth opportunity by paying an investment cost I_1 . For simplicity and to study the effect of debt overhang on the initial financing choice, this investment is financed with equity and increases the payoff by a factor of $\Pi > 1$. We assume that the growth opportunity is lost upon bankruptcy if it has not been used before. Absent from default, the debt is eventually redeemed and the firm is subsequently an all equity firm.

We analyze the optimal choice of the initial debt contract by studying each contract separately and then compare the initial firm value conditional on a given contract.

2.1 Convertible debt

Suppose the firm initially issues convertible debt. Denote the market value as $D_0^o(x)$ and let $D_1^o(x)$ be the debt value after the growth option has been exercised.⁶ The convertible option feature allows the debt holders to call the debt and receive a package specified in the debt contract.⁷ For simplicity we assume that debt can be converted into a known share, s, of the book value of equity. Naturally, the debt holders cannot be forced to supply new equity (s > 0), neither can they receive the hole firm (s < 1). We need to consider whether conversion takes place before or after the equity holders invest in the growth opportunity.

 $^{^{5}}$ To keep the analysis simple we only consider two types of debt contracts. A discussion on other types of contracts are included in Section 6 below

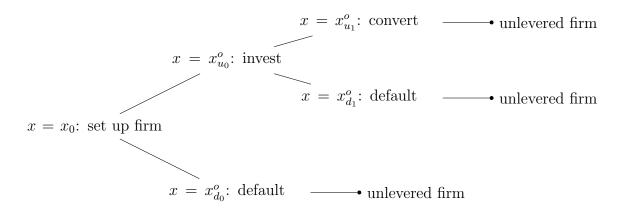
⁶For later use, the superscript "o" indicates that the debt is convertible, whereas the subscript "0" indicates that the growth option has not been exercised.

⁷There exist nine different conversion commodity types in the Mergent fixed income securities database (FISD): American depository shares, common stock, class A common stock, class B common stock, note/debenture, not available, purchase contract, preferred stock, and U.S. dollar. American depositary shares, common stock, class A common stock, class B common stock, and preferred shares comprise 99.18% of the conversion types in the database.

We focus on the latter case here.⁸ The time line is depicted in Figure 1.

Figure 1: Model set-up for at firm initially issuing convertible debt

This figure presents the time line for a firm that initially issued convertible debt. The firm is initially financed with debt and equity $(E_0, D_0^o = I_0)$. If earnings decreases to $x = x_{d_0}^o$, the firm defaults and continues as an unlevered firm $(E_1 = 0, D_1^o = (1 - \alpha)V_u)$. If earnings increases sufficiently the firm invests at $x = x_{u_0}^o$ $(E_0 = E_1 - I_1, D_0^o = D_1^o)$. Post-investment, if earnings increases to $x = x_{u_1}^o$ the debt is converted to equity and the firm continues as an unlevered firm $(E_2 = (1 - s)\Pi V_u, D_2^o = s\Pi V_u)$. If earnings decreases to $x = x_{d_1}^o$ the firm default and the firm continues as an unlevered firm $(E_2 = 0, D_2^o = (1 - \alpha)\Pi V_u)$.



The model is solved by backwards induction. For future reference, we denote the value of the unlevered firm as

$$V_u(x) = (1 - \tau) \frac{x}{r - \mu}.$$
 (2)

If the debt has been converted, the firm is an all-equity firm with an exercised growth option. Thus, the cash flow is Πx_t with present value

$$E_2^o(x) = \Pi V_u(x). \tag{3}$$

Suppose the firm has invested in the growth opportunity, but the debt holders have not converted their contract into equity. Then one of two things an happen. Either the cash flow increases enough inducing the debt holders to convert their claim. This happens the

⁸Section 6 consider the first case

first time cash flow reaches the level $x_{u_1}^o$. Another possible outcome is that the cash flow decreases so much that equity holders decide to default which occur when cash flow decreases to $x_{d_1}^o$. We assume the post-default firm stays unlevered, implying that the firm value is the reduced perpetual value of the existing production. This gives the following conditions:

$$D_1^o(x_{u_1}^o) = s E_2^o(x_{u_1}^o), (4)$$

$$E_1^o(x_{u_1}^o) = (1-s)E_2^o(x_{u_1}^o), (5)$$

$$D_1^o(x_{d_1}^o) = (1 - \alpha) E_2^o(x_{d_1}^o), \tag{6}$$

$$E_1^o(x_{d_1}^o) = 0. (7)$$

With convertible debt, the debt holders have an expost right to convert their claim to equity. They do so at a point in time which is optimal from their point of view. On the other hand, equity holders decide when to default. Therefore, the conversion trigger point $x_{u_1}^o$ and the default trigger $x_{d_1}^o$ are found by the standard smooth-pasting conditions

$$D_1^{o'}(x_{u_1}^o) = s E_2^{o'}(x_{u_1}^o), \tag{8}$$

$$E_1^{o'}(x_{d_1}^o) = 0. (9)$$

The next step is to derive the conditions for the debt value and the equity value before investment has taken place. Before investment, the equity holders default if cash flow decreases enough. Since investment increases the payoff by Π , we expect that $x_{d_0}^o > x_{d_1}^o$. If cash flow increases, investing in the growth opportunity becomes more attractive. Eventually, the value of waiting to invest is sufficiently low and the equity holders decide to invest. We assume this happens at $x_{u_0}^o$. The specific conditions for the debt value and the equity value at the respective boundaries are:

$$D_0^o(x_{u_0}^o) = D_1^o(x_{u_0}^o), (10)$$

$$E_0^o(x_{u_0}^o) = E_1^o(x_{u_0}^o) - I_1, (11)$$

$$D_0^o(x_{d_0}^o) = (1 - \alpha)(1 - \tau) \frac{x_{d_0}^o}{r - \mu},$$
(12)

$$E_0^o(x_{d_0}^o) = 0. (13)$$

At this stage in the model, it is the equity holders who have the right to either invest or to default. Therefore, the trigger for investment and default are found by solving the pair of smooth-pasting conditions

$$E_0'(x_{u_0}^o) = E_1'(x_{u_0}^o), \tag{14}$$

$$E_0'(x_{d_0}^o) = 0. (15)$$

The associated values for debt and equity are provided in Section 2.3 below.

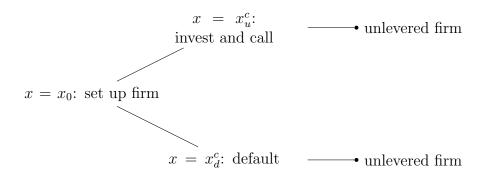
2.2 Callable debt

Now, suppose the firm initially issues callable debt. Again, the firm can invest in the growth option. Due to debt overhang, this investment has to be financed with new debt (see e.g., Julio, Julio; Frantz and Instefjord, 2019; Kruse et al., 2014). After investment, the firm continues as a pure equity firm. The time line is depicted in Figure 2.

The callable debt contract gives the firm the option to buy back the debt at a fixed price. In addition to the principal, the firm pays a premium which is the fraction p of the principal. The firm optimally call its debt when the underlying state variable becomes high enough. We denote this level of the state variable as x_u^c . Since a higher level of the cash flow also increases the present value of the investment opportunity, because the investment

Figure 2: Model set-up for at firm issuing callable debt

This figure presents the time line for a firm that initially issued callable debt. The firm is initially financed with debt and equity $(E_0^c, D_0^c = I_0)$. If earnings decreases to $x = x_d^c$, the firm defaults and continues as an unlevered firm $(E^c = 0, D^c = (1 - \alpha)V_u)$. If earnings increases sufficiently the firm invests at $x = x_u^c$, $(E^c = \Pi V_u - (1 + p)D^c - I_1, D^c = (1 + p)D^c(x_0))$ and continues as an unlevered firm.



cost, I_1 , is constant, the value of waiting to invest decreases in x. Thus, the firm eventually wants to invest. However, the existing debt induces a debt overhang problem (see e.g., Myers, 1977). If the equity holders infuse capital, this will mainly benefit the debt holders. Unless the cash flow is very high, and by that make debt almost risk-free, equity holders are not willing to provide capital to make the investment. On the other hand, new debt is precluded by covenants from the initial debt. Therefore, we assume that the firm invests in the growth opportunity at the same time it calls the initial debt. The alternative of waiting until existing debt is almost risk-free is too costly. Similar to before, after calling existing debt and investing, we assume that the firm continues as an all-equity firm. Thus, in our model framework, the tax shield is lost after the investment.⁹ Hence, the debt value and

⁹Obviously it would be beneficial for the firm to issue new debt, post-investment, to take advantage of the tax shield. However, to simplify the modeling framework, we do not consider this here. Our primary focus is to study the effects of choosing to issue either convertible or callable debt. With both types of debt contract, the firm continues as an all-equity firm. Therefore, we do not believe that this model limitation affects our main results.

equity value must at the call-investment threshold satisfy:

$$D^{c}(x_{u}^{c}) = (1+p)D^{c}(x_{0}), \tag{16}$$

$$E^{c}(x_{u}^{c}) = \Pi V_{u}(x_{u}^{c}) - (1+p)D^{c}(x_{0}) - I_{1}.$$
(17)

The equity holders call the debt when the smooth-pasting condition on the call boundary is satisfied:

$$E^{c'}(x_u^c) = \Pi V'_u(x_u^c).$$
(18)

Again, the cash flow can decreases so much that equity holders decide to default. Denote x_d^c the cash flow at default and the boundary conditions for debt and equity are:

$$D^{c}(x_{d}^{c}) = (1 - \alpha)V_{u}(x_{d}^{c}), \qquad (19)$$

$$E^c(x_d^c) = 0. (20)$$

The associated values for debt and equity are provided in Section 2.3 below.

2.3 Valuation of debt and equity

For expositional reasons it is convenient to first consider a general case. Eventually we want to valuate debt and equity, but let us for now consider a general claim F, which depends on the state variable x. Assume that when x reaches a high level, \overline{x} , the claim holder of \overline{F} receives the payment \overline{F} . Similarly, when x reaches a low level of the state variable, \underline{x} , the claim holder receives \underline{F} . Finally, we let the claim holder receive the payment flow $h_0 + h_1 x$ until either \underline{x} or \overline{x} is hit the first time. Note that \overline{F} , \underline{F} , h_0 , and h_1 are constants. **Proposition 1.** The present value of the claim F is

$$F(x) = h_0 + h_1 x + p_u(x; \overline{x})\overline{F} + p_d(x; \underline{x})\underline{F},$$
(21)

where

$$p_u(x;\bar{x}) = \frac{\underline{x}^{\beta_2} x^{\beta_1} - \underline{x}^{\beta_1} x^{\beta_2}}{\overline{x}^{\beta_1} x^{\beta_2} - x^{\beta_1} \overline{x}^{\beta_2}},$$
(22)

and

$$p_d(x;\overline{x}) = \frac{-\overline{x}^{\beta_2} x^{\beta_1} + \overline{x}^{\beta_1} x^{\beta_2}}{\overline{x}^{\beta_1} \underline{x}^{\beta_2} - \underline{x}^{\beta_1} \overline{x}^{\beta_2}}.$$
(23)

 $p_u(x; \overline{x})$ is the present value of receiving one unit of account at investment, conditional on not reaching the low state before, and $p_d(x; \overline{x})$ is the present value of receiving one unit of account at default, conditional on not reaching the high state before. With this proposition, we discuss the values for debt and equity given the initial debt contract.

Convertible debt

Suppose the firm issues convertible debt with conversion share $s \in (0, 1)$. Then, for a given coupon rate C^{o} , the "stage 0" value of debt is

$$D_0^o(x) = \frac{C^o}{r} + p_u(x; x_{u_0}^o) \left(D_1(x_{u_0}^o) - \frac{C^o}{r} \right) + p_d(x; x_{d_0}^o) \left((1 - \alpha) V_u(x_{d_0}^o) - \frac{C^o}{r} \right),$$
(24)

and the "stage 1" value of debt is

$$D_1^o(x) = \frac{C^o}{r} + p_u(x; x_{u_1}^o) \left(s \Pi V_u(x_{u_1}^o) - \frac{C^o}{r} \right) + p_d(x; x_{d_1}^o) \left((1 - \alpha) \Pi V_u(x_{d_1}^o) - \frac{C^o}{r} \right).$$
(25)

The first term of the stage-zero debt value in Equation (24) is the present value of receiving the coupon forever. The debt holders gets another value if investment or default occurs. If the firm invests, debt holders receive the stage-one value of debt and losses their current stream of coupon payments. $p_u(x; x_{u_0}^o)$ discounts this value to the present value. In case of default, debt holders receive the value of the unlevered firm less the cost of default. Conditional on not reaching investment before, $p_d(x; x_{d_0}^o)$ discount the value at default to the present value.

The stage-one value of debt is given in Equation (25). Post-investment, debt holders continue to receive coupon payments until they convert their claim or the firm defaults the first time. Upon conversion, debt holders give up their debt claim in exchange for a share, s, of the firm's equity. Since the firm subsequently has no debt, the debt holders thus receive a fraction of the unlevered firm times the investment scalar. $p_u(x; x_{u_1}^o)$ now discounts this value one stage back to stage-zero. In case of the default, debt holders receive the value of the unlevered firm times the investment scalar less the cost of default. Conditional on no conversion before, $p_d(x; x_{d_1}^o)$ discount the value at default to the stage-zero value.

The "stage 0" value of the equity claim is

$$E_0^o(x) = V_u(x) - \frac{C^o}{r} + p_u(x; x_{u_0}^o) \left(E_1(x_{u_0}^o) - I_1 - \left(V_u(x_{u_0}^o) - \frac{C^o}{r} \right) \right) - p_d(x; x_{d_0}^o) \left(V_u(x_{d_0}^o) - \frac{C^o}{r} \right)$$
(26)

and the "stage 1" value of equity is

$$E_1^o(x) = \Pi V_u(x) - \frac{C^o}{r} + p_u(x; x_{u_1}^o) \left((1-s) \Pi V_u(x_{u_1}^o) - \left(\Pi V_u(x_{u_1}^o) - \frac{C}{r} \right) \right) - p_d(x; x_{d_1}^o) \left(V_u(x_{d_1}^o) - \frac{C^o}{r} \right)$$
(27)

Before investment and default equity holders receive the value of the unlevered firm less the coupon payment to debt holders which correspond to the first term of the stage-zero value in Equation (26). If the firm invests, equity receive the stage-one value of equity, pay the investment cost and give up their current earnings. Again, $p_u(x; x_{u_0}^o)$ discounts this value to the present value. In case of default, equity gives up all value, and debt holders take over the firm. Conditional on not reaching investment before, again, $p_d(x; x_{d_0}^o)$ discount the value at default to the present value.

If earnings increase sufficiently such that the firm invests, Equation (27) presents the stage-one value function for equity. Post-investment the first term represents that equity holders receive the value of the unlevered firm times the investment scalar less the coupon payments until either conversion or default occurs. If earnings increases sufficiently such that conversion occurs, equity holders give up (1 - s) of their shares and the firm continues as an unlevered firm. Finally, the last term represents default at which equity gives up all value, and debt holders take over the firm.

Callable debt

Suppose the firm issues callable debt with call premium p and coupon rate C^c . This implies that the "stage 0" value of debt is

$$D_0^c(x) = \frac{C^c}{r} + p_u(x; x_u^c) \left((1+p)D^c(x_0) - \frac{C^c}{r} \right) + p_d(x; x_d^c) \left((1-\alpha)V_u(x_d^c) - \frac{C^c}{r} \right).$$
(28)

The first term of Equation 28 represents that debt holders receive the present value of the coupon payments until either the threshold for investment or the default boundary is hit for the first time. At-investment debt holders receive the principal and the call premium in exchange for their debt claim. This value is represented by the second term and discounted back to the present value via $p_u(x; x_u^c)$. In case of default, debt holders receive the value of the unlevered firm less the cost of default and lose the stream of coupon payments. Conditional on not reaching investment before, $p_d(x; x_d^c)$ discount the value to debt holders at default,

to the present value.

The value of the equity claim is

$$E_0^c(x) = V_u(x) - \frac{C^c}{r} + p_u(x; x_u^c) \left(\Pi V_u(x_u^c) - (1+p)D^c(x_0) - I_1 - \frac{C^c}{r} \right)$$
(29)

$$-p_d(x;x_d^c)\left(V_u(x_d^c) - \frac{C^c}{r}\right) \tag{30}$$

The first term of Equation (26) represents the value to equity holders before investment and default. This term is the value of the unlevered firm less the coupon payment to debt holder. If earnings increases to $x = x_u^c$ investment occurs. The second term represents the equity value upon investment. Equity holders pay the cost of investment and call the debt at a premium. Post-investment they receive the value of the unlevered firm times the investment scalar. In earnings decreases to $x = x_d^c$ the firm defaults. The last term represents that equity holders give up all value at default. Again, $p_u(x; x_u^c)$ and $p_d(x; x_d^c)$ discount the value of the equity claim at investment and default to the present value.

3 Debt overhang and debt repurchase

With our model set-up for the convertible and callable debt, we consider some model implications. To link this to our numerical and empirical analysis, we start off describing the first best investment policy and subsequently link this to the debt overhang problem. Furthermore, we study the timing of converting and calling the debt.

3.1 The effect of debt overhang

To understand how the different types of debt contracts affect the investment decision, first, we consider an all-equity firm. This firm's incentive for paying the investment cost I_1 is not disturbed by any outstanding debt. The firm invests at $x_{u,FB}$, where

$$x_{u,FB} = \frac{\beta_1}{\beta_1 - 1} \frac{I_1}{\Pi - 1} \frac{r - \mu}{1 - \tau}.$$
(31)

At the time of investment, the net present value is

$$NPV_{FB} = \frac{I_1}{\beta_1 - 1}.$$
 (32)

The investment threshold (31) and the at-investment net present value (32) provide useful intuition, which is also relevant for the case with debt. First, a more valuable growth opportunity (higher Π) expedites the time of investment. This is a standard result from capital budgeting using real options analysis; the effect stems from a lower value of waiting to invest. Intuitively, if the growth opportunity is sufficiently valuable, the firm should optimally invest immediately. Second, the value-added at investment is exactly offset by the earlier time of investment, and hence, the net present value does not depend on the scaling parameter.

In a similar vein, a higher investment cost I_1 incentivizes the firm to wait for a sufficiently high level of cash flow, which in turn increases the present value of the growth opportunity. Thus, a higher investment cost postpones the time of investment. However, in contrast to the scaling factor Π , the investment cost increases the at-investment net present value of the growth opportunity. This implies that a higher investment cost has two counterweighing effects. The direct, static, effect makes the investment less attractive because as a nowor-never decision, a higher investment cost decreases the growth opportunity's net present value. The indirect, dynamic effect increases the at-investment net present value, and by that mitigates the direct effect. These effects are particularly important when debt comes into play. When the firm has debt, the debt overhang problem discourages equity holders from undertaking the investment. The lower the net present value of the investment is, the worse is the debt overhang problem because of the wealth transfer from the equity holders to the debt holders increases. Thus, the direct effect of a higher investment cost amplifies the debt overhang problem. To mitigate this effect, the firm tends to issue a debt contract which the firm can control ex-post. That is, debt overhang tends to favor callable debt. On the other hand, the indirect effect of a higher investment cost increases the net present value of the growth opportunity at the time of investment. Furthermore, the present value of the firm's existing activities is also high, implying that the default risk of the outstanding debt is small. In turn, this decreases the wealth transfer to the debt holders at the time of investment. Thus, when the firm has the option to wait, a higher investment cost the debt overhang problem. This effect can dominate so that a higher investment cost rights to debt holders and, as a result, the firm prefers to issue convertible debt ex-ante. We elaborate further on this discussion in the numerical analysis below.

3.2 A simplified analysis of converting and calling debt

To gain further intuition about the difference between convertible debt and the callable debt, we consider the timing of conversion. That is when debt holders convert their debt as well as when equity holders call outstanding debt. Since we interested in the boundary for a high cash flow level, we abstract from the possibility to default. For simplification, we assume that the principal of callable debt is the perpetual value of receiving the coupon, C^c/r .

With these assumptions we obtain the threshold x_u^o for which debt holders convert their

debt

$$x_u^o = \frac{\beta_1}{\beta_1 - 1} \frac{C^o}{r} \frac{r - \mu}{1 - \tau} \frac{1}{s\Pi}.$$
(33)

The conversion threshold (33) intuitively depends on the core parameters. First, $\frac{\beta_1}{\beta_1-1} > 1$ is the standard scaling due to the value of the option to wait. Next, the higher the perpetual value of the coupon rate is, the higher must the value of the alternative (becoming a shareholder) be, which implies a higher level of the existing cash flow. Finally, a higher share or a more profitable growth opportunity make the alternative more valuable from the debt holders' point of view. Therefore, they are incentivized to convert their debt earlier.

Turning to callable debt, the threshold, x_u^c , for which the equity holders call the outstanding debt is

$$x_{u}^{c} = \frac{\beta_{1}}{\beta_{1} - 1} \left(p \frac{C^{c}}{r} + I_{1} \right) \frac{r - \mu}{1 - \tau} \frac{1}{\Pi - 1}.$$
 (34)

Again, $\frac{\beta_1}{\beta_1-1} > 1$ is the standard scaling due to the value of the option to wait. With callable debt, the alternative for is equity holders to call debt and invest. This alternative implies that equity holders must pay the call premium p and the cost of investment I_1 . In return, equity holders obtain the full claim on the underlying. Thus, equity holders call the debt only when the value of the underlying is higher than the cost of obtaining the full claim. Finally, a more profitable growth opportunity decreases the value of waiting.

4 Numerical analysis and model implications

To deepen our understanding of our model's implications, we provide a simulation. First, we describe our base case parametrization and the resulting debt contracts. Second, we discuss changes from the base case parametrization and relate these to empirical predictions. Below

we consider how our model's implications match our empirical findings.

4.1 Simulation procedure and variable definitions

For the implementation of the model, we use parameter values for our base case simulation presented in Tabel 1.¹⁰ The parameters follow previous literature (e.g., Christensen et al., 2014; Cooper, 2006; Hackbarth and Johnson, 2015).¹¹

Table 1: Parameters

This table presents our parameters for the base case simulation of the model. Our parameters follow previous literature (e.g., Christensen et al., 2014; Cooper, 2006; Hackbarth and Johnson, 2015).

	Parai	meter choices
Risk neutral drift of the EBIT process	μ	0
Volatility of the EBIT process	σ	0.2
Initial value of the EBIT process	x_0	0.3
Risk free interest rate	r	0.02
Tax rate	au	0.05
Bankruptcy costs	α	0.25
Investment scalar	П	2
Initial cost of setting up the firm	I_0	8
Cost of investment	I_1	6

With the base case parametrization, the optimal solution for convertible debt is presented in row one of Table 2 and for callable debt in row one of Table 3. For convertible debt, x_0^d is the value of the EBIT process for which the firm defaults before investment and x_1^d the default threshold after the investment. As expected, $x_1^d < x_0^d$, since the value-added from the investment increases the firm's payoff. Further, x_0^u is the investment threshold where the firm chooses to invest and finance the investment with equity. Finally, x_u^o is the value of the

¹⁰Setting $\mu > 0$ leads to similar results.

¹¹We note that a tax rate of 5% is low compared to previous literature, however, in our model framework τ is a measure for the tax advantage of debt. We discuss this assumption in Section 6.

earnings process for which the debt holders find it optimal to convert their claim. In that case, the equity holders give up the share amount, in column two, of their claim. Column one presents the optimal coupon. Columns three and four are the initial values of debt where the value of debt needs to equal or exceed the cost of setting up the firm, I_0 .

Table 2: Convertibel debt

This table presents the optimal coupon and share for convertible debt. The first row is using the base case parametrization in Table 1. Row two increases the investment cost $I_1 = 8$.

	Coupon						x_1^d	x_u^o
Base case	0.11	0.267	14.540	8.000	0.037	0.349	0.022	0.482
High I_1	0.11	0.274	13.150	8.006	0.039	0.458	0.022	0.470

For callable debt, the debt is called upon investment. Again, x_0^d is the default threshold before investment, and x_0^u represents the investment threshold. Columns one and two present the optimal coupon and call premium. Columns three and four are the initial values of equity and debt. Again, the value of debt must equal or exceed the cost of setting up the firm, I_0 .

Table 3: Callable debt

This table presents the optimal coupon and premium for callable debt. The first row is using the base case parametrization in Table 1. Row two increases the investment cost $I_1 = 8$.

Base case		Premium 0.274			0	$\frac{x_0^u}{0.503}$
High I_1	0.24	0.120	12.826	8.083	0.072	0.483

4.2 Numerical analysis

The first row of Table 2 and Table 3 presents the results from our base case simulation. We first discuss how debt overhang affects the initial financing choice. The results from our base case simulation and this discussion leads to a number of empirical predictions which we state subsequently. Section 6 considers further parameter variations.

The debt overhang problem

Recall that Section 3.1 provides a link between the investment cost and the debt overhang problem through a direct and an indirect effect. The direct effect of a higher investment cost worsens the debt overhang problem. To mitigate this effect, we expect firms to issue a debt contract which rants the firm ex-post control rights. Thus, the firm would choose callable debt. The indirect impact of a higher investment cost mitigates the debt overhang problem. Less debt overhang makes it cheaper for the firm to grant the ex-post right to debt holders, and thus the firm is more inclined to issue convertible debt. Hence, we test the effect of debt overhang by considering an increase in the cost of investment.

The second row of Table 2 and Table 3 present the results of increasing the cost of investment from $I_0 = 6$ to $I_0 = 8$. With both types of debt contracts, the increased cost of investment decreases the value of equity from 14.54 to 13.15 with convertible debt, and from 13.70 to 12.83 with callable debt. Thus, the relative decrease in equity value is smaller for callable compared to convertible debt (10% for convertible and 6% for callable). In turn, this suggests that a more severe debt overhang problem makes callable debt relative more valuable.

With convertible debt, the share increases compared to the base case (from 0.267 to 0.274). In turn, this incentivizes debt holders to convert their claim sooner. A comparison of x_u^o in row one and two show that debt is converted sooner with a higher cost of investment (from 0.349 to 0.458). Thus, our model predicts that if a firm is more exposed to debt overhang investors who hold convertible debt will convert their claim sooner. The same applies for callable debt. The threshold for repurchase decreases as the cost of investment increases (from 0.503 to 0.483). In general we therefor expect the holding time to be shorter for firms that are more exposed to the debt overhang problem.

4.3 Empirical predictions

From our simulation, several model implications arise. First, we consider the effect of debt overhang. From our discussion above, an increase in debt overhang decreases the value of the equity claim. The relative decrease in value is more substantial if the firm issues convertible rather than callable debt. Thus,

Hypothesis 1: Firms that are more exposed to debt overhang tend to issue callable rather than convertible bonds.

Second, from our simulation, we find that the optimal coupon is higher if the firm issues callable compared to convertible debt. Since holders of convertible debt have the option to convert the claim into equity, we would expect holders of callable debt to require a higher coupon as compensation. Thus,

Hypothesis 2: Firms issue callable bonds with a higher coupon compared to convertible bonds.

Finally, we consider the time between the initial issuance of a bond until it is either called or converted. With our base case parametrization, the conversion threshold for convertible debt, x_u^o , is lower than that of callable debt, x_u^c . Thus, the model suggests that the holding period for callable debt is longer than the holding period for convertible debt.

Hypothesis 3: Compared to the conversion of convertible bonds, callable bonds are held for longer period before they are called.

5 Empirical analysis

This section examines how our model's predictions correspond to empirical evidence, i.e., which factors influence the probability of issuing callable rather than convertible debt.

5.1 Data and descriptive statistics

Our data includes information on both firm and bond characteristics. We collect quarterly data on firms' accounting variables from Compustat. We exclude financials, utilities, and governmental firms (SIC codes 6000-6999, 4900-4999, and 9000-9999), as these firms are subject to different regulations. We collect the firms' bond information using the Mergent Fixed Income Securities Database (FISD) and merge it with the balance sheet data using firms' CUSIPs. We limit the analysis from 1990 to 2017, as the FISD database has limited coverage prior to that.

The information in the FISD database allows us to define whether a debt issue is convertible or callable. The database contains the flags "Convertible" and "Redeemable", indicating that the issue can be converted to common stock or another security of the issuer or that the bond is redeemable under certain circumstances. We define a convertible issue as a bond issue with a flag on convertible, no flag on redeemable, and for which the conversion commodity type is either "American Depository Shares" (ADS), "Common Stocks" (CS), "Class A Common Stock" (CSA), "Class B Common Stock" (CSB) or "Preferred Stock" (PS).¹² We define a callable bond as a bond with a flag on redeemable but no flag on convertible.¹³

From the bond information, we are mainly interested in the following variables: the bond's maturity, its coupon, its offering amount, and its holding time. The holding time is a measure for how long the investor holds the bond contract before it is either called or converted. The FISD database provides us with a schedule of future announced partial or full calls for every issue in the database. This variable allows us to define a flag for each of our convertible or callable issues, indicating whether the bond is converted or called. If the

¹²Other conversion commodity types include Note/Debenture (DEB), Not available (NA), Purchase contract (PC), U.S. Dollar (USD)

 $^{^{13}}$ We exclude all issues with flags on both convertible and redeemable, since our primary analysis focuses on the characteristics for either callable or convertible debt.

bond is convertible and "issue converted" is the action that took place to change the amount of outstanding we define this as a converted issue. We define a called issue as a callable bond for which the action type is "Balance of issue called", "Entire issues called", or "Part of an issue called". On average, 73% of the offering amount is called and 57% of the callable issues is called in full.¹⁴ We define the bonds holding period as the period between the issuance of the bond until the bond is converted or called.

To match the information on bond issues to the accounting data from Compustat, we recognize that each firm may issue several bonds in the same quarter. We want to determine the characteristics of a firm issuing one type of bond over the other. Therefore, we define firm-year quarters in which firm issues either only callable bonds or only convertible bonds. If multiple callable or convertible bonds are issued in the same quarter, the average of our variables of interest is used. Table 9 in Appendix provides a list of variable definitions.¹⁵ Beside the standard variables we also link the issuance of callable or convertible debt to a measure of debt overhang. We use the measure developed by Alanis et al. (2018), which represents the current value of lenders' rights to recoveries in default.¹⁶ One advantage of this measure is that it does not rely on the credit agencies' ability to assess firms' risk of default correctly. For every firm i and year t the debt overhang measure is given as

Debt overhang =
$$\frac{D_t}{K_t}$$
 · Recovery Rate · $\left[\sum_{s=1}^{20} \omega_t \left[1 - 0.05(s-1)\right](1+r)^{-s}\right]$. (35)

D represents the total debt and K refers to the capital stock. The recovery rate for defaulted senior unsecured bonds is from Altman and Kishore (1996), which groups defaulted bonds by SIC codes to account for the fact that the average prices at default vary between industries.

 $^{^{14}}$ Our modeling framework does not allow us to distinguish between full or partial calls. However, as we wish to study the timing of the option to call the debt contract we also consider partial calls.

¹⁵Our variables are defined according to the literature (e.g., Lemmon et al., 2008; Frank and Goyal, 2009; Leary and Roberts, 2010).

 $^{^{16}}$ We thank Alanis et al. (2018) for sharing this data.

Together, the first two terms of the debt overhang measure represents a measure for the lenders' recovery in default. The last term is a measure for the probability that the firm defaults, where a hazard model is used to forecast the default probabilities ω .¹⁷

The final sample comprises of 7,243 firm-quarters in which the firms issue callable bonds and 768 firm-quarters in which the firms issue convertible bonds. Table 4 Panel A presents descriptive statistics for the quarters in which the firms issue callable bonds and Panel B for quarters in which the firms issue convertible debt. In general, the median values of maturity and coupon are 9.49 and 6.15.¹⁸ They are significantly higher for callable bonds than for convertible bonds (9.70 and 7.00 vs. 5.03 and 3.00, respectively). Additionally, the median holding time of a callable bond is also larger than the one of a convertible bond (2.55 years vs. 1.05 years). Firms that issue callable bonds tend to have more leverage, a higher return on assets, less cash, and more tangible assets. Further, they are larger in size, are less overvalued (measured by Tobin's Q), and with a lower mean of debt overhang.

The results regarding the coupon payments align with our model predictions. In our model framework, holders of callable bonds receive a higher coupon compared to holders of convertible bonds. As the holder of a callable bond have no embedded option they are compensated by a higher coupon payment. Our empirical analysis supports this hypothesis. The summary statistics also suggests that the holding time of a callable bond is longer than that of a convertible. This was also the case for our base-case numerical implementation of the model.

¹⁷This model allows Alanis et al. to estimate default probabilities for each firm-year, without relying on the availability of a bond rating. The hazard model has been shown to outperform other bankruptcy predicting model. Previous literature such as Hennessy et al. (2007) uses he Moody's hazard rate of default. However, this restricts the sample of firms for which we can compute a measure of debt overhang. Furthermore, credit rating based default measures assign the same default probability to all firms within a credit rating class. They do so even though investors may not perceive them as having the same credit risk.

 $^{^{18}\}mathrm{Table}$ 10 depicts the summary statistics for our overall sample of bonds

Table 4: Summary Statistics

This table presents summary statistics for our sample of callable and convertible bond issues. Panel A presents descriptive statistics for firm year quarters in which callable bonds are issued. Panel B presents descriptive statistics for firm year quarters in which convertible bonds are issued. The sample period covers 1990 to 2017 with variable definitions in the Appendix 9.

	Panel A: Callable						
	count	mean	median	sd	p10	p90	
Maturity	7,243	10.844	9.701	7.600	5.425	19.922	
Offering amount	$7,\!243$	12.609	12.612	0.847	11.608	13.605	
Coupon	7,243	7.031	7.000	2.939	3.138	10.875	
Holding time	$7,\!243$	3.499	2.553	3.655	0.000	9.310	
Leverage	7,071	0.434	0.389	0.242	0.192	0.715	
Return on assets	$6,\!823$	0.033	0.033	0.029	0.011	0.060	
Cash	$7,\!065$	0.091	0.048	0.121	0.006	0.217	
Dividend repurchase	$6,\!608$	0.021	0.001	0.082	0.000	0.053	
Tangibility	7,017	0.366	0.307	0.263	0.063	0.790	
Size	7,071	8.239	8.172	1.688	6.062	10.464	
Tobin's Q	$6,\!351$	1.718	1.451	0.923	1.024	2.658	
Debt overhang	4,810	0.051	0.003	1.543	0.001	0.042	

		1	and D. C	Onvertie	nc	
	count	mean	median	sd	p10	p90
Maturity	768	6.021	5.026	3.808	3.762	7.055
Offering amount	768	12.261	12.206	0.945	11.156	13.528
Coupon	768	3.323	3.000	2.375	0.750	6.250
Holding time	768	2.307	1.049	2.381	0.000	5.729
Leverage	765	0.399	0.352	0.264	0.160	0.680
Return on assets	719	-0.010	0.0185	0.457	-0.045	0.0479
Cash	764	0.307	0.231	0.258	0.027	0.711
Dividend repurchase	663	0.023	0.000	0.058	0.000	0.081
Tangibility	763	0.203	0.110	0.214	0.021	0.575
Size	765	7.149	7.002	1.419	5.528	9.131
Tobin's Q	760	2.579	1.8131	3.531	1.013	4.791
Debt overhang	580	0.190	0.006	2.932	0.000	0.089

Panel B: Convertible

5.2 Empirical strategy

We use the following probit regression to estimate the probability of issuing callable debt rather than convertible debt:

$$Pr(call)_{i,t} = \theta_0 + \gamma_1 \;(\text{maturity})_{t,i} + \gamma_2 \;(\text{offering amount})_{t,i} + \gamma_3 \;(\text{coupon})_{t,i} + \gamma_4 \;(\text{holding time})_{t,i} + \gamma_5 \;(\text{debt overhang})_{t,i} + \beta \;X_{i,t} + \alpha_y + \alpha_i + \varepsilon_{i,t},$$
(36)

where Pr(call) is equal to one if the bond issuance was callable and zero if it was convertible. The vector X includes seven firm-specific control variables following the literature.¹⁹ α_y and α_i are year and industry fixed effects and the standard errors are clustered at the firm level.

We subsequently examine several cross-sectional sample splits by estimating the above regression for different sub-samples. We examine differences in (1) the amount of debt overhang a firm faces, (2) the size, and (3), differences in the economic environment, i.e., crisis vs. non-crisis years.

5.3 Empirical results

Before discussing how bond characteristics and debt overhand influence the decision to issue a callable or convertible bond, let us review some of the firm characteristics that influence this decision. Echoing the previous literature (e.g., Kish and Livingston, 1992; Robak and Kish, 2000), the results in Table 5 show that bigger firms with a higher return on assets are associated with a higher probability of issuing callable debt, whereas, firms with more cash are more likely to issue convertible bonds.

 $^{^{19}}$ These are: 1) leverage, 2) return on assets, 3) cash, 4) dividend repurchases, 5) tangibility, 6) firm size, and 7) Tobin's Q.

5.3.1 All firms

We first analyze how our main coefficients relate to the probability of issuing a callable bond. Model (1) in Table 5 relates the probability of issuing a callable bond to bond characteristics only. Model (2) adds firm characteristics, and model (3) further includes the measure of debt overhang. The first column for each model presents the coefficients from the probit regression while the second column reports the average discrete change.

Our results from the base case regression in Table 5 confirms our second and third hypotheses. The issuance of callable bonds is positively related to the bond's coupon and the time between issuance and action date of the bond. Increasing the coupon by one standard deviation on average increases the probability of issuing a callable bond 7.2%. Furthermore, there is a significant positive relationship between the issuance of a callable bond and the bond's maturity. This finding is similar to Becker, Campello, Thell, and Yan (Becker et al.). However, this paper focus only on callable bonds and does not consider convertible bonds. We find a positive correlation between the issuance of a callable bond and offering amount. Thus, compared to convertible bonds, we expect callable bonds to have a longer maturity and a higher offering amount.

Adding firm characteristics, column 2 shows that bigger and more profitable firms are more likely to issue callable bonds, whereas firms with higher cash levels are more likely to issue convertible bonds. Increasing the cash by one standard deviation on average decreases the probability of issuing a callable bond 1.8%. Jensen (1986) rationalize the use of convertible debt to avoid free cash flow problems. In this vein, we would expect firms with higher cash holdings to be more likely to issue convertible debt.

Model (3) supports our first hypothesis by showing that there is a positive relation between debt overhang and the probability of issuing a callable rather than convertible bond. A one standard deviation increase in the debt overhang measure increases the probability of

Table 5: Base case regressions

This table presents the results of our baseline probit regression. We regress the probability of issuing a callable bond (relative to issuing a convertible bond) on bond characteristics as well as firm characteristics. In model one we include bond characteristics, model two further includes firm characteristics, and finally, model three also include the measure of debt overhang. Column two, four, and six presents the average discrete change. We present the variable definitions in the Appendix 9, and the measure for debt overhang follows Alanis et al. (2018). All models include year and industry fixed effects and standard errors are clustered at the firm level. The symbols ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

	Model	1	Mode	12	Mode	13
	Coefficient	ADC	Coefficient	ADC	Coefficient	ADC
Maturity	$\begin{array}{c} 0.134^{***} \\ (7.11) \end{array}$	0.062	$\begin{array}{c} 0.134^{***} \\ (6.30) \end{array}$	0.051	$\begin{array}{c} 0.130^{***} \\ (6.06) \end{array}$	0.047
Offering amt	$\begin{array}{c} 0.774^{***} \\ (9.87) \end{array}$	0.049	$\begin{array}{c} 0.271^{***} \\ (3.28) \end{array}$	0.015	$\begin{array}{c} 0.283^{***} \\ (2.99) \end{array}$	0.015
Coupon	$\begin{array}{c} 0.365^{***} \\ (16.59) \end{array}$	0.072	$\begin{array}{c} 0.421^{***} \\ (14.75) \end{array}$	0.063	$\begin{array}{c} 0.435^{***} \\ (12.95) \end{array}$	0.062
Holding time	$\begin{array}{c} 0.051^{***} \\ (2.74) \end{array}$	0.015	0.040 (1.85)	0.009	0.046^{**} (2.17)	0.010
Leverage			0.230 (1.02)	0.004	$-0.202 \\ (-0.91)$	-0.003
Return on assets			$ \begin{array}{c} 6.340^{***} \\ (5.40) \end{array} $	0.057	7.579^{***} (5.89)	0.018
Cash			-1.523^{***} (-5.44)	-0.018	-1.689^{***} (-4.78)	-0.020
Div repurchase			$1.021 \\ (1.17)$	0.005	0.814 (0.83)	0.004
Tangibility			-0.053 (-0.29)	-0.001	-0.234 (-1.02)	-0.004
Size			$\begin{array}{c} 0.444^{***} \\ (8.45) \end{array}$	0.043	0.450^{***} (7.37)	0.041
Tobin's q			0.090^{**} (2.33)	0.010	$0.045 \\ (1.09)$	0.003
Debt overhang					0.021^{*} (1.70)	0.003
Year FE	Y		Y		Y	
Industry FE	Y		Y		Y	
N Adj R^2	$6,060 \\ 0.535$		$5,56 \\ 0.63$		$4,22 \\ 0.65$	

issuing a callable bond 3%. This suggests that to mitigate the effect of debt overhang, the firms tend to issue a debt contract which ex post can be controlled by the firm.

5.3.2 Cross-sectional sample splits

Next, we examine how the issuance of callable and convertible bonds relates to differences in the amount of debt overhang, the size of the firm, and differences in economic environments. These cross-sectional splits allow us to obtain further intuition regarding which of the effects from our modelling framework predominates in the empirical data.

Debt overhang

First, we consider the effect of firms having above or below median debt overhang. Table 6 reports the results. The coefficient on offering amount is only significant for firms with abovemedian debt overhang and not for firms with below. Hence, firms that are more exposed to debt overhang appear to issue callable bonds with higher offering amounts. From the base case regression, we know that more debt overhang increases the likelihood of a firm issuing callable rather than convertible bonds. In line with our first hypothesis, this is related to the fact that with more debt overhang, firms are more likely to issue debt contracts which can ex post be controlled by the firm.

The coefficient on holding time is only significant for firms with below median debt overhang. This is in line with our discussion from the numerical analyses. Our model predicts that the holding time for callable and convertible bonds are shorter if the firm is exposed to debt overhang. Our empirical analysis suggests that firms that are more exposed to the debt overhang problem does not hold their callable bonds significantly longer than the convertibles.

Table 6: Split on debt overhang

This table presents the results of a probit regression where we split our sample according to the median of the measure of debt overhang. We regress the probability of issuing a callable bond (relative to issuing a convertible bond) on bond characteristics as well as firm characteristics for each subsample. Model one represents firms with above median debt overhang and model two firms with below. Column one and three presents the coefficients from the probit regression. Column two and four presents the average discrete change. We present the variable definitions in the Appendix 9, and the measure for debt overhang follows Alanis et al. (2018). All models include year and industry fixed effects and standard errors are clustered at the firm level. The symbols ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

	Model	1	Model	2	
	Coefficient	ADC	Coefficient	ADC	
Maturity	0.146^{***}	0.049	0.141^{***}	0.046	
	(4.25)		(4.81)		
Offering amount	0.388***	0.025	0.343	0.014	
	(3.54)		(1.60)		
Coupon	0.455***	0.087	0.544^{***}	0.053	
	(11.49)		(9.06)		
Holding time	0.035	0.008	0.081^{**}	0.015	
	(1.37)		(2.13)		
Debt overhang $>$ median	Y		Ν		
Debt overhang $<$ median	Ν		Υ		
Constants	Y		Y		
Year FE	Y Y				
Industry FE	Υ		Y		
Adj. R^2	0.674	:	0.673		
(N)	1,793		1,968		

Table 7 splits our sample on firm size. The coefficient on maturity is significantly bigger for large firms compared to small firms. This is in line with the literature arguing that larger firms issue debt with longer maturity (e.g., Barclay and Smith, 1995; Stohs and Mauer, 1996). The intuition is that the issuing of public debt has a considerable fixed cost. Large firms can take advantage of significant scale economies, whereas smaller firms cannot. We find that both small and large firms issue callable debt with a longer maturity compared to convertible debt. However, for large firms, the difference in maturity is even more pronounced. Thus, in our sample, larger firms issue debt with longer maturity.

The effect of the offered amount of debt differs between large and small firms. Columns 1 and 2 show that large firms issue callable debt with a lower offering amount, whereas small firms issue callable debt with a higher offering amount. In general the mean amount offered is higher (632, 998 million USD) for large compared to small firms (356, 193 million USD). This aligns with the findings in Kurshev and Strebulaev (2015) who demonstrate that the relationship between leverage and size is positive. Again, fixed costs of financing contribute to the explanation of the stylized size-leverage relationship.

The mean offering amount of convertible for large firms is 891, 952 million USD compared to 623, 502 million USD for callable bonds. For smaller firms the median for convertible is 283, 412 million USD and 387, 558 million USD for callable. Thus, larger firms are more likely to issue larger amounts of convertible debt compared to smaller firms. We conjecture that larger firms which are less exposed to debt overhang would be even more likely to issue more convertible debt. In fact large firms with below median debt overhang issues convertible debt with a mean offering amount of 850, 976 million USD and callable with a mean of 637, 850 million USD. Smaller firms with above median debt overhang issues convertible debt with a mean offering amount of 233, 116 million USD and callable with an offering amount of

Table 7: Split on size

This table presents the results of a probit regression where we split our sample according to the median of the measure of firm size. We regress the probability of issuing a callable bond (relative to issuing a convertible bond) on bond characteristics as well as firm characteristics for each subsample. Model one represents firms with above median size and model two firms with below. Column one and three presents the coefficients from the probit regression. Column two and four presents the average discrete change. We present the variable definitions in the Appendix 9, and the measure for debt overhang follows Alanis et al. (2018). All models include year and industry fixed effects and standard errors are clustered at the firm level. The symbols ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

	Mode	l 1	Model	2	
	Coefficient	ADC	Coefficient	ADC	
Maturity	$\begin{array}{c} 0.370^{***} \\ (4.79) \end{array}$	0.055	$\begin{array}{c} 0.0979^{***} \\ (3.92) \end{array}$	0.025	
Offering amount	-0.568^{***} (-3.44)	-0.022	$\begin{array}{c} 0.653^{***} \\ (3.12) \end{array}$	0.032	
Coupon	$\begin{array}{c} 0.344^{***} \\ (5.84) \end{array}$	0.030	$\begin{array}{c} 0.585^{***} \\ (12.86) \end{array}$	0.112	
Holding time	0.082^{*} (1.85)	0.013	0.063^{*} (1.95)	0.012	
Size > median	Y		N		
Size $<$ median	Ν		Y		
Controlds	Y		Y		
Year FE	Y Y				
Industry FE	Y		Y		
Adj. R^2	0.60	3	0.748		
(N)	1,58	4	1,937		

311,607 million USD. Again firms which are more exposed to the debt overhang problem tend to issue more callable debt. On the other hand, larger firms that are less exposed to debt overhang does not find debt which can be controlled by the firm ex post as valuable. Instead, they issue convertible debt and therefore face lower coupon payments.

Our results supports our second hypothesis by showing that both small and large firms issue callable debt with a coupon which is higher than for convertible debt. Again, holders of callable debt do not hold an embedded option. Instead, they are compensated by a higher coupon. The effect on the coupon is higher for small firms. We expect small firms to face a higher cost of debt and henceforth issue debt with a higher coupon. Similar, Yu (2005) argues that larger firms tend to have a smaller default risk and, hence, a lower cost of debt financing. Our data confirms this intuition.

Crisis vs. non-crisis periods

To study how the issuance of callable debt is related to economic environment, Table 8 differentiates between firm-quarters within a financial crisis and firm-quarters not within a crisis period.²⁰ The results in Table 8 show that in crisis periods the coefficient on maturity is higher compared to non-crisis periods. In both economic environments callable bonds are issued with a longer maturity compared to convertible bonds. However, in economic downturns, callable bonds are issued with a longer maturity compared to non-crisis periods. This suggests that in crisis periods firms do not wish to roll over debt as frequently.

Only in non-crisis periods, the holding time of a callable bond is significantly longer than that of a convertible. In our total sample of callable and convertible bonds issues in the period between 1990-2017, 2% of the bonds are called and 0.4% is converted during time of crisis. In comparison, 25% is called and 3% converted in non-crisis periods. During crisis

²⁰We use the crisis/non-crisis quarters as defined by the NBER. Crisis quarters are 1990q3–1991q1, 2001q1–2001q4, 2007q4–2009q2. Out of total of 8,011 callable or convertible bond issues, 842 are issued during a crisis period.

Table 8: Split on crisis

This table presents the results of a probit regression where we split our sample according to financial crises. We use the crisis/non-crisis quarters as defined by the NBER. Crisis quarters are 1990q3–1991q1, 2001q1–2001q4, 2007q4–2009q2. We regress the probability of issuing a callable bond (relative to issuing a convertible bond) on bond characteristics as well as firm characteristics for each subsample. Model one represents firms in crisis quarters and model two firms in no-crisis quarters. Column one and three presents the coefficients from the probit regression. Column two and four presents the average discrete change. We present the variable definitions in the Appendix 9, and the measure for debt overhang follows Alanis et al. (2018). All models include year and industry fixed effects and standard errors are clustered at the firm level. The symbols ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

	N. 1. 1. 1		M 110		
	Model 1		Model 2		
	Coefficient	ADC	Coefficient	ADC	
Maturity	0.263***	0.071	0.112^{***}	0.040	
	(3.78)		(4.75)		
Offering amount	0.544^{**}	0.030	0.196***	0.010	
	(2.17)		(2.60)		
Coupon	0.352***	0.053	0.479^{***}	0.063	
	(5.30)		(13.89)		
Holding time	0.040	0.009	0.0515^{**}	0.011	
	(0.55)		(2.33)		
Crisis	Y		N		
Controls	Y		Y		
Year FE	Y Y				
Industry FE	Y		Y		
Adj. R^2	0.598		0.660		
(N)	560		3,871		

periods firms make fewer investments (Duchin et al., 2010) which implies that they do not call their debt to undertake as many investments. Further, it could be more costly to issue new debt in economic downturns. Thus, firms do not call existing debt to issue new.

The coefficient on offering amount is positive in both economic environments but higher in crisis periods. Thus, in economic downturns, firms issue callable debt with higher offerings compared to convertible debt. We conjecture that the option to call debt, to finance e.g. investments, are more valuable for the firms in crisis periods. Thus, we expect firms to issue more callable debt when the economic conditions are less favorable.

Again, we find support for our second hypothesis. Callable debt is issued with a higher coupon than convertible, however, the effect is larger in non-crisis periods. During economic downturns financial distress is more likely. If a firm is in financial distress, the option to convert a debt claim into equity is less valuable for investors. Thus, the difference between the coupon payments on callable and convertible bonds should be smaller.

6 Discussions and possible extensions

In this section, we briefly discuss additional model implications which lead to further empirical predictions. We also briefly discuss some possible model extensions.

6.1 Additional model implications

Tables 11 and 12 present an extended parameter variation for our numerical analyses. Besides the model implications presented above, our framework offers several other testable implications related to the holding time of callable and convertible bonds. The third rows of Tables 11 and Table 3 consider the effect of changing the initial value of the earnings process, x_0 . We decrease the value to $x_0 = 0.25$ such that, compared to the base case, the firm now faces lower initial earnings. A lower initial earnings stream decreases the threshold for calling and converting the debt claim (from 0.503 to 0.473 and from 0.482 to 0.377). However, when we take the initial value of the earnings process into account, the holding time of convertible bonds is decreased (from 0.132 to 0.127). In contrast, we have an increase in the holding time of the callable bond (from 0.203 to 0.223).

The conversion threshold is increasing in the coupon and decreasing in the share (Equation (33)). Compared to the base case, there is no change in coupon but the share increases from 0.267 to 0.332. The investment threshold for callable debt is increasing in the coupon and premium (Equation (34)). Both are increased compared to the base case. Thus, our framework predicts that the holding time of convertible bonds decreases and the holding time of callable bonds increases if a firm has lower initial earnings.

The risk-free interest rate

In row eight of Table 2 and Table 3 we increase the discount rate to r = 2.5%. For convertible debt, it postpones the investment (from 0.349 to 0.412) and the time of conversion (from 0.482 to 0.687). For callable debt, it decreases the time to conversion (from 0.503 to 0.478). For callable debt, the optimal premium is lower compared to the base case (from 0.274 to 0.198) which from lowers the threshold for conversion (Equation (34)). For convertible debt, the increase in optimal share (from 0.267 to 0.3) is not enough to counter the effects of the increased discount rate (Equation (33)). Thus, with a higher discount rate, our framework predicts that the holding time is longer for convertible compared to callable bonds.

The investment scalar

In the last row of Tables 11 and 12 we consider the effect of increasing the investment scalar to $\Pi = 2.5$. A higher investment scalar makes the investment opportunity more profitable, and we consider how this affects the conversion thresholds. With convertible debt, this implies that the investment threshold decreases from 0.349 to 0.249. Further, for convertible debt, the increase in investment scalar implies a higher coupon (from 0.11 to 0.16) but lower share (from 0.267 to 0.1). The conversion threshold is increasing in the coupon and decreasing in the share (Equation (33)). The low share dominates the increase in coupon such that the conversion threshold increases (from 0.482 to 1.658). With callable debt, the coupon decreases (from 0.20 to 0.18). The premium on callable debt increases from 0.27 to 0.52. The investment threshold for callable debt is increasing in the coupon and premium and decreasing in the investment scalar (Equation (34)). Since x_0^u increases from 0.503 to 0.512, the first two effects dominates the latter. Thus, in both cases, the predicts that a more profitable investment opportunity increases the holding time for both types of debt contracts.

6.2 Possible model extensions

Conversion threshold

To keep our model framework as simple as possible, we assume that the conversion of convertible debt takes place after the investment. One possible model extension is to consider the effect of allowing the holder of convertible debt to convert before the investment. In this case, debt holders become equity holders before the investment and therefore, would have to pay for the investment. From the expression of the threshold for conversion in Equation (33) we see that a higher share or a more profitable growth opportunity incentivize debt holders to convert their claim earlier. If conversion before investment can occur, debt holders need to balance the benefits of early conversion against the cost of investment and the lost coupon payments.

The issuance of new debt

Another simplifying assumption is the fact that we do not allow the firm to issue new debt after the initial debt has been either called or converted. One implication of this assumption is that the firm, at some point, cannot take advantage of the tax-shield. From our numerical analysis (rows 5 of Table 11 and 12), we see that for a high tax-advantage of debt, it was not optimal to convert the convertible debt claim. If the firm could issue a new debt contract, we expect that conversion of convertible debt would be optimal even with a higher tax advantage of debt.

Risk of preemption

The investment opportunity in our model framework is perpetual. Thus, there is no risk of losing the option to invest. We conjecture that if we included the threat of preemption, this would affect the choice between the issuance of convertible or callable debt. The risk of preemption implies that the firm can only invest for a short period. Thus, preemption risk amplifies the debt overhang problem. To the extent that the debt overhang problem is increases, we conjecture that preemption risk increases the value of callable relative to convertible debt.

7 Conclusion

Using a dynamic model, we examine the choice between issuing callable or convertible debt. Our model predicts that firms issue callable debt with a higher coupon compared to convertible debt. Further, the time between the issuance of callable debt until it is called is longer than the time between issuance and conversion for convertible bonds. Data on corporate bond issues in the U.S confirm these predictions. In general, callable bonds are issued with a higher coupon, are held for a longer time, have longer maturity, and a higher offering amount compared to convertible bonds. Also, firms that are more exposed to debt overhang tend to prefer callable over convertible debt. These results support our intuition that, with debt overhang, firms are more likely to issue debt contracts which can ex-post be controlled by the firm.

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8 Appendix

8.1 Proof of Proposition 1

Proof. We first derive the value of a general claim, F, which depends on x as a state variable. In addition, the claim hold of F receives the payment \overline{F} when x reaches a high level of the state variable, \overline{x} . Similarly, \underline{F} is received when x reaches a low level of the state variable, \underline{x} . Finally, let the claim holder receive the payment flow $h_0 + h_1 x$ until either \underline{x} or \overline{x} is hit the first time. Since the dynamics of x follows (1) and the required rate of return on any claim is the risk-free rate of return, standard arguments give us that the claim F must satisfy the ordinary differential equation (see e.g., Dixit and Pindyck, 1994; Hackbarth and Mauer, 2012; Christensen et al., 2014)

$$\frac{1}{2}\sigma^2 x^2 F''(x) + \mu x F'(x) - rF(x) + h_1(x) + h_0 = 0.$$
(37)

Given this we also have that

$$F(x) = \frac{h_1 x}{r - \mu} + \frac{h_0}{r} + f_1 x^{\beta_1} + f_2 x^{\beta_2}, \qquad (38)$$

where the coefficients f_1 and f_2 are to be found below. Note that the assumption $\mu < r$ comes into play here. The powers, β_i , solve the quadratic equation

$$\frac{1}{2}\sigma^2\beta_i(\beta_i - 1) + \mu\beta_i - r = 0,$$
(39)

with $\beta_1 > 1$ and $\beta_2 < 0$. To find the coefficients f_1 and f_2 we use the value matching conditions to get two equations:

$$\frac{h_1\overline{x}}{r-\mu} + \frac{h_0}{r} + f_1\overline{x}^{\beta_1} + f_2\overline{x}^{\beta_2} = \overline{F},\tag{40}$$

$$\frac{h_1 \underline{x}}{r - \mu} + \frac{h_0}{r} + f_1 \underline{x}^{\beta_1} + f_2 \underline{x}^{\beta_2} = \underline{F}$$
(41)

Linear algebra gives us that the solution on vector form is

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \frac{1}{\overline{x}^{\beta_1} \underline{x}^{\beta_2} - \underline{x}^{\beta_1} \overline{x}^{\beta_2}} \begin{pmatrix} \underline{x}^{\beta_2} & -\overline{x}^{\beta_2} \\ -\underline{x}^{\beta_1} & \overline{x}^{\beta_1} \end{pmatrix} \begin{pmatrix} \overline{F} - \frac{h_1 \overline{x}}{r-\mu} - \frac{h_0}{r} \\ \underline{F} - \frac{h_1 \underline{x}}{r-\mu} - \frac{h_0}{r} \end{pmatrix}.$$
 (42)

The present value claim that gives one unit of account, if x hits \overline{x} before \underline{x} , can thus be derived setting $h_0 = h_1 = \underline{F} = 0$ and $\overline{F} = 1$. We readily get

$$p_u(x;\bar{x}) = \frac{\underline{x}^{\beta_2} x^{\beta_1} - \underline{x}^{\beta_1} x^{\beta_2}}{\overline{x}^{\beta_1} \underline{x}^{\beta_2} - \underline{x}^{\beta_1} \overline{x}^{\beta_2}},$$
(43)

and similarly we get

$$p_d(x;\overline{x}) = \frac{-\overline{x}^{\beta_2} x^{\beta_1} + \overline{x}^{\beta_1} x^{\beta_2}}{\overline{x}^{\beta_1} \underline{x}^{\beta_2} - \underline{x}^{\beta_1} \overline{x}^{\beta_2}}.$$
(44)

Using (43) and (44) and the relevant expressions for h_0, h_1, \underline{F} , and \overline{F} for debt and equity, respectively, it is easy to get the debt value (24) and the equity value (26).

8.2 Tables

Table 9: Variable definitions

This table presents variable definitions. Our variables are defined according to the literature (e.g., Lemmon et al., 2008; Frank and Goyal, 2009; Leary and Roberts, 2010). We use the measure of debt overhang developed by Alanis et al. (2018). Accounting data is from Compustat and we exclude financials, utilities, and governmental firms (SIC codes 6000-6999, 4900-4999, and 9000-9999), as these firms are subject to different regulations. We collect the firms' bond information using the Mergent Fixed Income Securities Database (FISD) and merge it with the balance sheet data using firms' CUSIPs. We limit the analysis from 1990 to 2017, as the FISD database has limited coverage prior to that.

Variable	Definition
Maturity	Maturity date - Offering date
Holding time	Effective date - Offering date
Coupon	coupon
Offering amount	log(off_amt)
Total debt	short term debt $(dlcq) + long term debt (dlttq)$
Leverage	Total debt / book assets (atq)
Return on assets	operating income before depreciation (oibdpq) / book assets
Firm size	log(book assets)
Dividend repurchase	preferred dividends (dvpq)
	+ purchase of common and preferred (prstck) / book assets
Market equity	Price close (prccq) * Common shares (cshprq)
Market-to-book	(Market equity + Total debt) / book assets
Tobin's q	(Book assets + (Common shares outstanding (cshoq) * Price close)
	- Common equity (ceqq)) / Book assets
Investment	Capital Expenditures (capx) / Book assets

Table 10: Summary Statistics

This table presents summary statistics for our entire sample. The sample period covers 1990 to 2017 with variable definitions in the Appendix 9.

	Full sample						
	count	mean	median	sd	p10	p90	
Maturity	9,534	10.83	9.49	7.63	5.01	20.03	
Offering amount	$9,\!534$	12.50	12.55	0.90	11.51	13.56	
Coupon	$9,\!534$	6.24	6.15	3.16	2.11	10.50	
Holding time	$9,\!534$	3.36	2.58	3.45	0.00	8.19	
Leverage	$9,\!353$	0.42	0.38	0.24	0.18	0.70	
Return on assets	8,945	0.03	0.03	0.13	-0.00	0.06	
Cash	$9,\!345$	0.14	0.06	0.18	0.01	0.37	
Dividend repurchase	$8,\!577$	0.02	0.00	0.08	0.00	0.05	
Tangibility	9,292	0.33	0.26	0.26	0.05	0.75	
Size	$9,\!353$	7.94	7.81	1.71	5.81	10.25	
Tobin's Q	8,610	1.91	1.52	1.62	1.03	3.03	
Debt overhang	6,700	0.07	0.00	1.63	0.00	0.05	

Table 11: Convertibel debt

This table presents the optimal coupon and share for convertible debt. Each row represents a different parametrization. The first row is using the base case parametrization in Table 1. Row two increases the investment cost $I_1 = 8$. In the third row we decrease the initial value of the earnings process to $x_0 = 0.25$. In row four, α is increased to 0.3. In the fifth row, the tax rate is increased to $\tau = 0.2$. In row six, the growth rate of the earnings stream is increased to $\mu = 0.01$ and in row seven the volatility to $\sigma = 0.22$. In the eight row, the discount rate is increased to r = 0.025 and in the last row the investment scalar is increased to $\Pi = 2.5$.

	Coupon	Share	Equity	Debt	x_0^d	x_0^u	x_1^d	x_u^o
Base case	0.11	0.267	14.540	8.000	0.037	0.349	0.022	0.482
High I_1	0.11	0.274	13.150	8.006	0.039	0.458	0.022	0.470
Low x_0	0.11	0.332	9.983	8.003	0.039	0.350	0.022	0.377
High α	0.09	0.280	14.525	8.003	0.0314	0.345	0.018	0.373
High τ	0.22	0.010	11.290	8.009	0.067	0.437	0.0428	36.045
High μ	0.17	0.010	43.264	8.008	0.0258	0.3012	0.019	20.430
High σ	0.09	0.280	14.609	8.008	0.029	0.377	0.016	0.395
High r	0.19	0.300	8.896	8.008	0.070	0.412	0.041	0.687
High Π	0.16	0.100	22.390	8.004	0.044	0.249	0.025	1.658

Table 12: Callable debt

This table presents the optimal coupon and premium for callable debt. Each row represent a different parametrization. The first row is using the base case parametrization in Table 1. Row two increases the investment cost $I_1 = 8$. In the third row we decrease the initial value of the earnings process to $x_0 = 0.25$. In row four, α is increased to 0.3. In the fifth row, the tax rate is increased to $\tau = 0.2$. In row six, the growth rate of the earnings stream is increased to $\mu = 0.01$ and in row seven the volatility to $\sigma = 0.22$. In the eight row, the discount rate is increased to r = 0.025 and in the last row the investment scalar is increased to $\Pi = 2.5$.

	Coupon	Premium	Equity	Debt	x_0^d	x_0^u
	-		1 0		<u> </u>	<u> </u>
Base case	0.20	0.274	13.700	8.117	0.061	0.503
High I_1	0.24	0.120	12.826	8.083	0.072	0.483
Low x_0	0.23	0.315	9.342	8.042	0.069	0.473
High α	0.20	0.280	13.699	8.079	0.061	0.503
High τ	0.27	0.070	10.448	8.002	0.078	0.464
High μ	0.16	0.090	42.260	8.380	0.0247	0.400
High σ	0.22	0.220	14.090	8.021	0.060	0.500
High r	0.29	0.198	8.400	8.099	0.095	0.478
High Π	0.18	0.519	17.270	9.640	0.052	0.512